spin coherent state:

$$|z_1, z_2\rangle_j = \sum_{N_1+N_2=N} \sqrt{\frac{N!}{N_1!\,N_2!}}\; z_1^{N_1} z_2^{N_2}\; |j, m\rangle$$

$\left(\frac{1}{2j+1}\right)$

$$\to \sqrt{\frac{(2j)!}{(j+m)!\,(j-m)!}}\;(z_1)^{j+m}\,(z_2)^{j-m}\;|j,m\rangle \qquad \frac{(j+m)!\,(j-m)!}{(2j+1)!}$$

$$(z_1)^{2j}\,(z_1)^{m-j}\,(z_2)^{j-m}\,|j,m\rangle$$

$$= (z_1)^{2j} \sum_{m=-j}^{m=j} \left(\frac{z_2}{z_1}\right)^{j-m} |j,m\rangle \qquad \to \frac{\Gamma(j+m+1)\,\Gamma(j-m+1)}{\Gamma(2j+2)}$$

$$\cdot \int d\Omega_{SU(2)}\, |z_1, z_2\rangle\langle z_1, z_2| = \int d\Omega_{SU(2)} \sum_{n, m=-j}^{n, m=j} (\bar{z}_1)^{j+m} (\bar{z}_2)^{j-m}$$

$$(z_1)^{j-n}\,(z_2)^{j+n} \sqrt{\frac{(2j)!}{(j+m)!\,(j-m)!}} \cdot \sqrt{\frac{(2j)!}{(j+n)!\,(j-n)!}}\; |j,m\rangle\langle j,n|$$

$$= \sum_{m=-j}^{m=j} \int d\Omega_{SU(2)}\, (\bar{z}_1 z_1)^{j+m}\,(\bar{z}_2 z_2)^{j-m}\, \frac{(2j)!}{(j+m)!\,(j-m)!}\; |j,m\rangle\langle j,m|$$

$$= \sum_{m=-j}^{m=j} \int_0^{2\pi} d\beta_1 \int_0^{2\pi} d\beta_2\, \frac{1}{2\pi^2}\, 2\int_0^{\pi/2} \cos\theta \cdot \sin\theta \cdot (\cos\theta)^{2(j+m)}\,(\sin\theta)^{2(j-m)}$$

$$d\theta \cdot \frac{(2j)!}{(j+m)!\,(j-m)!}\; |j,m\rangle\langle j,m| \qquad \begin{cases} a = j+m+1 \\ b = j-m+1 \end{cases}$$

$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}\,dx = 2\int_0^1 (\sin\theta)^{2(a-1)}\,(\cos^2\theta)^{2(b-1)}\,\sin\theta\cos\theta\, d\theta$$

$$= \int_0^1 (\sin\theta)^{2a-1}\,(\cos\theta)^{2b-1}\,d\theta = \frac{\Gamma\left(\frac{j+m}{2}+1\right)\Gamma\left(\frac{j-m}{2}+1\right)}{\Gamma(2j+2)}$$

$$d\Omega_{S_3} = \frac{1}{\pi^2}\, dz_1 d\bar{z}_1\, dz_2 d\bar{z}_2$$

Volume

$$dz_1 = \cos\theta\, e^{i\beta_1} i\, d\beta_1 - \sin\theta\, e^{i\beta_1} d\theta$$

$$d\bar{z}_1 = -\cos\theta\, e^{-i\beta_1} i\, d\beta_1 + - \sin\theta\, e^{-i\beta_1} d\theta$$

$$S_3 = \frac{2\pi\Gamma\left(\frac{d}{2}\right)}{\pi^{d/2}} \quad : \quad d = 2$$

$$\cancel{S_n = \frac{\pi^{n/2}}{\Gamma\left(\frac{n}{2}+1\right)}} \qquad \cancel{S_2 = \frac{\pi^{1}}{\Gamma}}$$

$$dz_1 d\bar{z}_1 = \cos^2\theta\, d\beta_1^2 + \sin^2\theta\, d\theta^2$$

$$S_n = \frac{2\pi^{n/2}}{\Gamma\left(\frac{n}{2}\right)} \qquad \left\{ S_3 = \frac{2\pi^{1.5}}{\Gamma\left(\frac{3}{2}\right)} = 4\pi^2 \right.$$

$$dz_2 d\bar{z}_2 = \cancel{\sin}\ \cancel{\cos^2\theta}\ \sin^2\theta\, d\beta_1^2 + \cos\theta\, d\theta$$

$$\int_0^1 4\pi^2 r^3\, dr = \pi^2 \quad : \quad \underline{Vol(R^3) = \pi^2}$$

$\underline{\det g_{ij}}$ $\underline{dz_1}$

$$\left\{ \begin{aligned} \bar{z}_1 &= \cos\theta\, e^{i\beta_1} \\ z_2 &= \cos\theta\, e^{i\beta_2} \end{aligned} \right. \qquad \begin{aligned} \bar{z}_1 &= \cos\theta\, e^{-i\beta_1} \\ \bar{z}_2 &= \cos\theta\, e^{i\beta_2} \end{aligned}$$

$$z_1 = \cancel{\cos\theta}\, r\, e^{i\theta}$$

$$\bar{z}_1 = \cancel{\cos\theta}\, r\, e^{-i\theta}$$

$$\left\{ \begin{aligned} dz_1 &= dr\, e^{i\theta} + i\cdot r e^{i\theta} d\theta \\ d\bar{z}_1 &= dr\, e^{-i\theta} - i r e^{-i\theta} d\theta \end{aligned} \right.$$

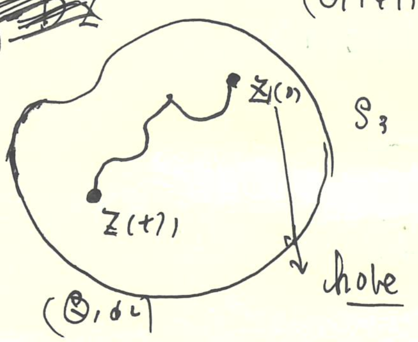
$$dz_1 d\bar{z}_1 = (dr)^2 + r^2 d\theta^2$$

$\underline{\sin\theta\cos\theta}$

metric : $\underline{\cos^2\theta\, d\beta_1^2 + \sin^2\theta\, d\beta_1^2 + d\theta^2} = ds^2$

Berry phase:

$$\langle z_f^{(t)} | \; | z_{(0)} \rangle = \prod_{n'=1}^{n} \langle z_{n+1} | z_n \rangle \langle z_n | \cdots \cdots \langle z_1 | z_0 \rangle$$



$\downarrow \; SU(2) \otimes SU(2)$

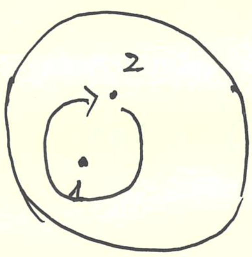
coherent

two spin coherent states copies

$\boxed{SU(2) \otimes SU(2)}$

写下 particle 和 hole 激发的波函数

$$\Psi \simeq \Psi_1^{j_1+m_1} \Psi_2^{j_1-m_2} \Psi_3^{j_2+m_2} \Psi_4^{j_2-m_2} e^{-\bar{\Psi}\Psi}$$



$\boxed{\text{Non-abelian Berry phase:}}$

measurmemet of spin coherent state (SU(2))

$$(\vec{n}' \cdot \vec{J}) | n(\theta, \phi) \rangle = s | \theta, \phi \rangle^{\vec{n}'}$$

$$\langle \vec{n}_1' | \vec{n}_2' \rangle = \left(\frac{1 + \vec{n}_1 \cdot \vec{n}_2}{2} \right)^s$$

$$\langle z_1', z_2' | z_1, z_2 \rangle = \langle \cancel{j, m} | \sum_{m=-j}^{j} \langle j, m | \frac{(2j)!}{(j+m)!(j-m)!} (\bar{z}_1' z_1)^{j+m} (z_2' \ldots)^{j-m} | j, m \rangle$$

$$\langle n_2 | \vec{n}_1 \cdot \vec{J} | n_1 \rangle = S \langle n_2 | n_1 \rangle$$

$$|\vec{n}_1\rangle = \frac{1}{(1+z^2)^j} \exp(zJ_-)|j,j\rangle$$

$$= \frac{1}{(1+z^2)^j} \sum_{m=-j}^{j} \sqrt{\frac{(2j)!}{(j+m)!(j-m)!}}\, z^{j-m} |j,m\rangle$$

$$\vec{n}\cdot\vec{J} = \frac{1}{2}\left(z\frac{\partial}{\partial z} - \bar{z}\frac{\partial}{\partial \bar{z}} \right) \qquad \left(1 + \frac{u_1\bar{u}_2}{v_1\bar{v}_2}\right)^{2j}$$

spin coheret 态非把: vacam 换成 $|j,m\rangle$ $\qquad (v_1\bar{v}_2 + u_1\bar{u}_2)^{2j}$

$$|\vec{n}_1\rangle = \sum_{m=-j}^{j} \binom{2j}{j+m} (u_1)^{j+m} (v_1)^{j-m} |j,m\rangle$$

$$\langle \vec{n}_2 | \vec{n}_1 \rangle = \sum_{m=-j}^{j} \binom{2j}{j+m}^2 \underline{(u_1\bar{u}_2)^{j+m} (v_1\bar{v}_2)^{j-m}}$$

$$= \sum_{m=-j}^{j} \binom{2j}{j+m}^2 (u_1\bar{u}_2 v_1\bar{v}_2)^j \left(\frac{u_1\bar{u}_2}{v_1\bar{v}_2}\right)^m \qquad m = m-j$$

$$= \sum_{m=0} \binom{2j}{m} (u_1\bar{u}_2 v_1\bar{v}_2)^j \left(\frac{u_1\bar{u}_2}{v_1\bar{v}_2}\right)^{-j} \left(\frac{u_1\bar{u}_2}{v_1\bar{v}_2}\right)^m$$

$$= \sum_{m=0}^{2j} \binom{2j}{m} (v_1\bar{v}_2)^{2j} \left(\frac{u_1\bar{u}_2}{v_1 v_2}\right)^m$$

- $$(u_1 \bar{u}_2 + v_1 \bar{v}_2)^{2j} = \frac{(1+\bar{z}_2 z_1)^{2j}}{(1+|z_2|^2)^j (1+|z_1|^2)^j}$$

- $$|f\rangle = \sum_{m=-j}^{j} \binom{2j}{j+m}^{1/2} \left(\cos\frac{\theta}{2}\right)^{j+m} \left(\sin\frac{\theta}{2}\right)^{j-m} e^{i(j-m)\phi} |j,m\rangle$$

$$\langle f'|f\rangle = \sum_{m=-j}^{j} \binom{2j}{j+m} \left(\cos\frac{\theta}{2}\cos\frac{\theta'}{2}\right)^{j+m} \left(\sin\frac{\theta}{2}\sin\frac{\theta'}{2}\right)^{j-m}$$

$$= \left(\sin\frac{\theta}{2}\sin\frac{\theta'}{2}\right)^{-2j} \sum_{m=-j}^{j} \binom{2j}{j+m} \left(\cos\frac{\theta}{2}\cos\frac{\theta'}{2} \Big/ \sin\frac{\theta}{2}\sin\frac{\theta'}{2}\right)^{j+m}$$

$$= \left(\sin\frac{\theta}{2}\sin\frac{\theta'}{2}\right)^{-2j} \sum_{m=0}^{2j} \binom{2j}{m} \left(\cot\frac{\theta}{2}\cot\frac{\theta'}{2}\right)^{m}$$

$$= \left(\sin\frac{\theta}{2}\sin\frac{\theta'}{2}\right)^{-2j} \left(1 + \cot\frac{\theta}{2}\cot\frac{\theta'}{2}\right)^{2j}$$

$$= \left(\sin\frac{\theta}{2}\sin\frac{\theta'}{2} + \cos\frac{\theta}{2}\cos\frac{\theta'}{2}\right)^{2j}$$

$\cos\theta\cos\theta'$

$\Uparrow$

$\sin\theta\sin\theta'\cos(\phi-\phi')$

$\Longrightarrow$ $\sin\theta\sin\theta' e^{i(\phi-\phi')} + \sin\theta\sin\theta' e^{i(\phi'-\phi)}$

$$= \left(\sin^2\frac{\theta}{2}\sin^2\frac{\theta'}{2} + \frac{1}{2}\sin\theta\sin\theta' + \cos^2\frac{\theta}{2}\cos^2\frac{\theta'}{2}\right)^j$$

$\hat{n}_1\cdot\hat{n}_2 = \cos\theta\cdot\cos\theta' + \sin\theta\cdot\sin\theta'\cos(\phi-\phi')$

$$\left[\left(\frac{1-\cos\theta}{2}\right)\left(\frac{1-\cos\theta'}{2}\right) + \frac{\sin\theta\sin\theta'}{2} + \left(\frac{1+\cos\theta}{2}\right)\left(\frac{1+\cos\theta'}{2}\right)\right]^j$$

$$= \left[\frac{1}{2} + \frac{\sin\theta\sin\theta' + \cos\theta\cos\theta'}{2}\right]^j$$

$2(xx' + yy') = (x+iy)(x'-iy') + (x-iy)(x'+iy')$

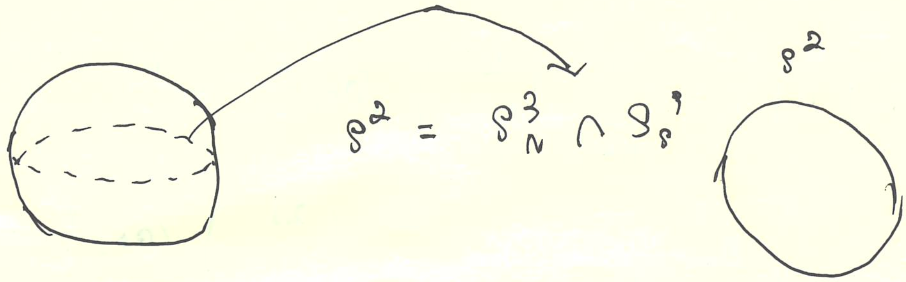
$$\left\{\begin{array}{l} x+iy = \sin\theta e^{i\phi} \\ x-iy = \sin\theta e^{-i\phi} \end{array}\right. \Longrightarrow$$

$$|\bar{u}_i' u_i + \bar{v}_i' v_i|^2 = \left(\cos\frac{\theta'}{2} \cos\frac{\theta}{2} e^{i(\phi-\phi')} + \sin\frac{\theta'}{2} \sin\frac{\theta}{2} e^{-i(\phi-\phi')} \right)^2$$

$$(u_i' \bar{u}_i + v_i' \bar{v}_i)(u_i \bar{u}_i' + v_i \bar{v}_i') = \sin\theta \sin\theta' \cos(\phi-\phi')$$

$$+ \cos\theta \cdot \cos\theta'$$

$\cancel{F' \subset S^2}$



$$S^2 = S^3_N \cap S^1_s$$

SU(1,1) coherent states : Perelomov

$$SU(1,1) \xrightarrow{\text{real copy}} SU(2,\mathbb{R})$$

illustration : $P_+ = \{ z \in \mathbb{C},\ \mathrm{Re}\, z > 0 \}$

Mobius transformation:

$$P_+ \ni \theta \cdot z \longmapsto Z = e^{i\phi} \frac{z - z_0}{z - \bar{z}_0} \in D$$

SU(3) : Def: $T^a = a_i^+ \lambda^a_{ij} a_j$

$$[T^a, T^b] = \cancel{i\epsilon_{abc}}\, i\, \cancel{\epsilon_{abc}} f_{abc} T^c$$

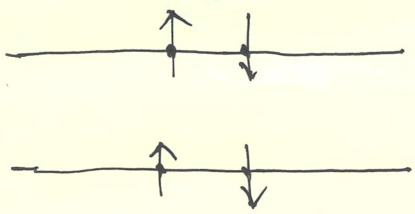
$$\Rightarrow \quad \underline{T_3 = a_1^+ a_1 - a_2^+ a_2}$$

$$\underline{T_8 = a_1^+ a_1 + a_2^+ a_2 - a_3^+ a_3}$$

$\boxed{\text{polynomials}}$

$$\underline{\langle \cancel{(m_1),(m_2)} \,|\, V \,|\, (m_1),(m_2) \rangle}$$

1. How to construct (4+1D) tranverse Ising model:



$\boxed{\text{4个 Flavor 还是两个？}}$

$$n^a = (\Psi^+_\uparrow, \Psi^+_\downarrow)\, \sigma^a\, (\Psi_\uparrow, \Psi_\downarrow)$$

还是

$$n^a = (\Psi^+_{3/2}, \ldots\ldots \Psi^+_{-3/2})\, T^a\, \Psi$$

2. State operator correspondence

SO(5) symmetry

particle hole symmetry

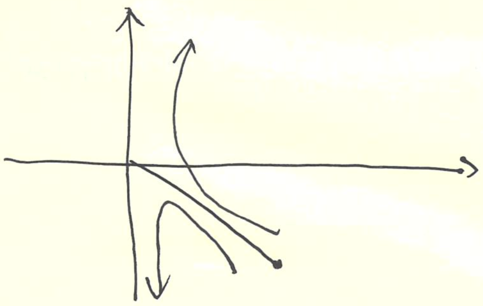
4.

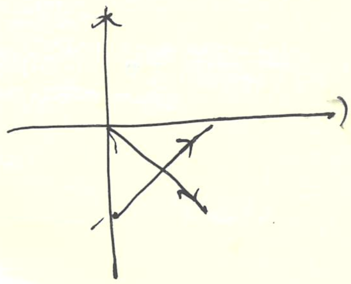
? ? ?

$SO(5) \supset SO(4) \supset SO(3) \supset SO(2)$

对称性~~变~~降低到更低的时候：，这个时候会看到什么

5: 只有 ϕ^4 理论的时 λ 才是 irrelevent





6. $\lambda^a =$

$\lambda^1 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & 0 \end{pmatrix}$ $\lambda^2 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & 0 \end{pmatrix}$ $\lambda_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & 0 \end{pmatrix}$

$\lambda^4 = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_1 \end{pmatrix}$ $\lambda^5 = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_2 \end{pmatrix}$ $\lambda_8 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$

$\lambda^6 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ $\lambda^7 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & i \end{pmatrix}$

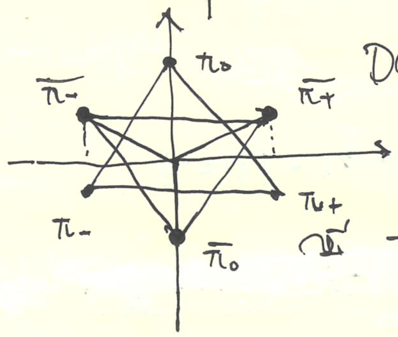
$$\begin{cases} I = \sum z_i \lambda^3_{ij} \dfrac{\partial}{\partial z_j} = z_1 \dfrac{\partial}{\partial z_1} - z_2 \dfrac{\partial}{\partial z_2} = p_1 - p_2 \\ Y = z_1 \dfrac{\partial}{\partial z_1} + z_2 \dfrac{\partial}{\partial z_2} - 2 z_3 \dfrac{\partial}{\partial z_3} = p_1 + p_2 - 2p_3 \end{cases}$$

$$\Phi = z_1^{p^1} z_2^{p^2} z_3^{p^3}$$

$$\Downarrow$$

Fundamental irrep

(1, 0)

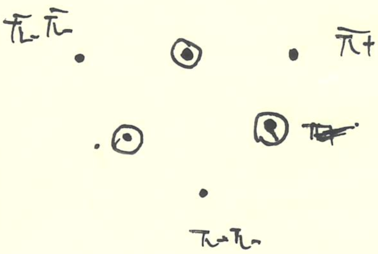


$$D(p) = (S+1)(S+2)/2$$

$$\Phi \to e^{ip\theta} \Phi$$

(1, 0) $\boxed{\pi_0}$ $\boxed{\pi_-}$ $\boxed{\pi_+}$

(0, 1) $\boxed{\bar{\pi}_0}$ $\boxed{\bar{\pi}_-}$ $\boxed{\bar{\pi}_+}$



$\pi_+\pi_+ \to \boxed{\pi_+|\pi_+}$

~~(2, 0)~~

为什么没有反对称表示？

因为只有U(1)的性

(2, 0) $\boxed{|}$

monopole charge $\longrightarrow$? S

$$\frac{S^3}{U(1)} = S^2$$

$\downarrow$ gauge field

$Y = 2(Q - I_3)$

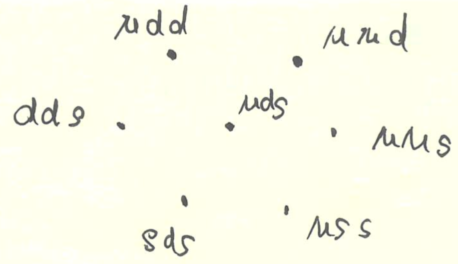
Quark	Isospin	hypercharge	charge
u	$1/2$	$\frac{1}{3}$	$2/3$
d	$-1/2$	$\frac{1}{3}$	$-1/3$
s	0	$-\frac{2}{3}$	$-1/3$

$$\Rightarrow \quad p_1 - p_2 = 1 \quad : \quad p_1 + p_2 - p_3 = -4$$

$$\cancel{\Psi(2l+6) = z_1^{l+3}\, z_2^{l+1}\, z_3^{2l+5/2} \quad : \quad s}$$

$$\begin{cases} Y = \frac{1}{3}(n_d + n_u - 2n_s) \\ I = \frac{1}{2}(n_u - n_d) \end{cases}$$

*	u	d	s
$S = 1$:	z_1	z_2	
$S = 2$			z_3^2
$S = 3$	$z_1^2 z_2$		


udd
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uss

$$\begin{pmatrix} x_5 & x_0 - i\sigma_i \\ x_0 + i\sigma_i & -x_5 \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

$$\Psi_+^{(1)} = \begin{pmatrix} \cos\frac{\theta}{2} \\ 0 \\ \sin\frac{\theta}{2}\sin\frac{\beta}{2} e^{-i(\alpha+\gamma)} \\ i\sin\frac{\theta}{2}\sin\frac{\beta}{2} e^{i(\alpha-\gamma)} \end{pmatrix}$$

$$\cancel{(i\sigma_1)(i\sigma_2) = (i)^2 \sigma_1\sigma_2 =}$$

$$q_i \left\{ \begin{array}{l} i \longrightarrow i\sigma_x \\ j \longrightarrow i\sigma_y \\ k \longrightarrow -i\sigma_z \end{array} \right. \Longrightarrow \cancel{ij = -ji = k}$$

$$\Psi_+^{(2)} = \begin{pmatrix} \cos\frac{\theta}{2}\sin\frac{\beta}{2} e^{i(\alpha+\gamma)} \\ -i\sin\frac{\theta}{2}\sin\frac{\beta}{2} e^{i(\alpha-\gamma)} \\ \sin\frac{\theta}{2} \\ 0 \end{pmatrix}$$

$$P_+ = \frac{1}{2}\begin{pmatrix} x_5 + 1 & x_0 - x_i q_i \\ x_0 + x_i q_i & 1 - x_5 \end{pmatrix} \begin{pmatrix} \Psi_1 \\ 0 \end{pmatrix}$$

$\cancel{1 + \cos\theta}$

$$x_0 - ix_3 = \sin\theta \cos\frac{\beta}{2} e^{-i(\alpha+\gamma)} \quad 2\cos^2\frac{\theta}{2}$$

$$x_1 + ix_2 = \sin\theta \cos\frac{\beta}{2} e^{i(\alpha-\gamma)}$$

$$\cancel{\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = 2\sqrt{1+x_5}}$$

$$P_+ e_1 = \frac{1}{\sqrt{2(1+x_5)}} \begin{pmatrix} (1+x_5)\Psi_1 \\ (x_0 + x_i q_i)\Psi_2 \end{pmatrix} = \frac{1}{\sqrt{2(1+x_5)}} \begin{pmatrix} 1 + x_5 \\ 0 \\ x_0 - ix_3 \\ i(x_1 + ix_2) \end{pmatrix}$$

$\Psi_+^{(1)}$

$$\begin{pmatrix} x_0 - ix_3 & i(x_1 - ix_2) \\ i(x_1 + ix_2) & x_0 + ix_3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$P_+ e_3 = \frac{1}{\sqrt{2(1-x_5)}} \begin{pmatrix} (x_0 - ix_i q_i)\Psi_2 \\ (1 - x_5)\Psi_2 \end{pmatrix} = \frac{1}{\sqrt{2(1-x_5)}} \begin{pmatrix} x_0 + ix_3 \\ -i(x_1 + ix_2) \\ 1 - x_5 \\ 0 \end{pmatrix}$$

$$\Psi = \begin{pmatrix} \cancel{x_5}+1 & -(x_4 - x_i q_i) \\ -(x_4 + x_i q_i) & 1 + x_5 \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

$$P_- e_2 = \begin{pmatrix} 0 \\ 1 - x_5 \\ -i(x_1 - ix_2) \\ -(x_4 - ix_3) \end{pmatrix} \qquad \begin{pmatrix} x_4 + ix_3 & i(x_1 + ix_2) \\ i(x_1 + ix_2) & x_4 - ix_3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\searrow i(x_3 + ix_4)$ $\quad r \longrightarrow \Sigma$

$$\rightarrow \begin{pmatrix} 0 \\ \sin\frac{\theta}{2} \\ -i\cos\frac{\theta}{2}\sin\frac{\beta}{2} e^{-i(\alpha \pm \gamma)} \\ \cancel{-i}\cos\frac{\theta}{2}\cos\frac{\beta}{2} e^{-i(\alpha \pm \gamma)} \end{pmatrix}$$

$$P_1 e_2 = \begin{pmatrix} i(x_1 - ix_2) \\ -i(x_3 + ix_4) \\ 0 \\ 1 + x_5 \end{pmatrix} \qquad - \begin{pmatrix} x_4 - ix_3 & -i(x_1 - ix_2) \\ -i(x_1 + ix_2) & x_4 + ix_3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} x_1 - ix_2 \\ -(x_4 + ix_3) \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} i\sin\frac{\theta}{2} \cancel{\sin}\cancel{\cos\beta} \sin\frac{\beta}{2} e^{-i(\alpha+\gamma)} \\ -i\sin\frac{\theta}{2}\cos\frac{\beta}{2} e^{i(\alpha+\gamma)} \\ 0 \\ \cos\frac{\theta}{2} \end{pmatrix}$$

2粒

$$e^{i\alpha S_z} \left(\begin{array}{c|c} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} e^{-i\phi} \\ \hline \sin\frac{\theta}{2} e^{i\phi} & \cos\frac{\theta}{2} \end{array} \right) e^{-i\gamma S_z}$$